## The Functions of Mathematics Education and Differentiation in the Mathematics Teaching as an Opportunity of Education for Everyone

Klimova E. N.<sup>1</sup>

Germany, Schwäbisch Gmünd, University of Education

Functions of mathematics education are integrated by the idea of unity of the material (acquisition of knowledge that will be used in practice) and formal (the development of thinking and personality in the process of acquiring knowledge) approaches. The objective of didactics of mathematics is the creation and analysis of the learning environments, which possess and realizes the potential for the material and formal education components for every child [1, P. 22]. Competency model of educational standards in mathematics (see fig. 1, cf. [1, P. 12] and [2, P. 11]) reflects basic components of mathematics education in Germany in secondary school. The substantial mathematical competencies are structured into five major concepts: Algorithm and number, Space and form, Measuring, Variable and functional relation, Data and randomness. Along with these competencies, the competencies connected with mathematical process (Skill-competencies) are singled out into a special dimension.



Figure 1. A Knowledge-Skill-Competencies Model in Mathematics

<sup>&</sup>lt;sup>1</sup> <u>elena.klimova@ph-gmuend.de</u>, +49 71 71-953-457



Figure 2. Differentiation in the school education system

Individuality of pupils and creating opportunities for harmonious education of every child necessitates differentiated approach to teaching. Significant external differentiation takes place in Germany at the end of the fourth form as a result of the separation of school-children into different types of schools (see fig. 2, cf. [3, P. 105]). In this case, gymnasiums are essentially different from other types of schools by granting admission to higher education (Germ. Abitur).

Differences in the level of competencies of schoolchildren (these levels are reflected in the scale Level of complexity in the competency model of fig. 1), depending on the type of schools are clearly seen in the results of PISA-Test, see. Fig. 3. However, the same results indica te that even within schools of one type, study groups are very heterogeneous. Strong students from real school exceed the level of competencies of average students from gymnasiums. Teaching in heterogeneous study groups is a serious current didactic problem. At the same time the external differentiation at such an early age (after 4th form) remains the object of didactic discussions. In Baden-Württemberg, as an alternative to external differentiation, a concept of united school (Germ. Gemeinschaftsschule), e.g., is offered, which is currently being actively developed (42 schools in 2012/13 school year, 87 in 2013/14, 129 in 2014/15, see [5]). The concept of such a school gives preference to internal differentiation and involves co-education during the first 10 school years.

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Figure 3. Results of PISA-Test in Germany. Figure by author, based on [4]

Mathematics teaching the classes, which in any case are very heterogeneous, certainly requires special approaches to content, methods and forms of education, as well as additional teachers' competencies. The so-called internal cellular differentiation in mathematics classes requires, in addition to existing and actively used methods, the development of such tasks as: tasks with a stepwise level of requirements, parallel tasks, self-differentiating tasks, etc. For example, all schoolchildren are able to solve the problem of what could be the air volume in the balloon (see Fig. 4) or explore the properties of the sum of three consecutive natural numbers can all students, but each at their own level, depending on their level of competencies. In the ideal case of well thought-out and organized internal differentiation, the development of mathematical competencies of pupils occurs in an optimal way according to their initial level and abilities (see fig. 5).



Figure 4.



*Figure 5.* The concept of internal differentiation. Figure by author, based on [6]

## **List of references**

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